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MATH TUTORIAL

Q1. $f(x) = x^5 - 5x + 2 = 0$

$x_0 = 1$

$x_1 = 2$

Ans. $f(1) = -2 < 0$

$f(2) = 24 > 0$

First approx., $(n=1)$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{(1)(24) - (2)(-2)}{24 + 2}$$

$$= \frac{24 + 6}{27} = \frac{30}{27} = 1.0769$$

$\cancel{f(1.1111) = -1.8620}$

$f(\underset{x_2}{1.0769}) = -1.9361$

Second app., $(n=2)$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{2(-1.9361) - 1.0769(24)}{-1.9361 - 24}$$

$$= 1.1458$$

$f(x_3) = f(1.1458) = -1.7541$

Third app., $(n=3)$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

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$$x_4 = \frac{1.0769(-1.7541) - 1.1458(-1.9561)}{(-1.7541) + (1.9561)}$$

$= 1.8098$

$f(x_4) = 12.328$

Fourth app.,(n=4)

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(1.1458)(12.328) - (1.8098)(-1.7541)}{12.328 - 1.1458}$$

$= 1.547$

$f(x_5) = 3.125$

Fifth app., (n=5)

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

~~= 1.4576~~

$$\frac{(1.809)(3.125) - (1.547)(12.328)}{3.125 - 12.328}$$

~~f(x6) =~~

~~Sixth app. (n=6~~

$= 1.458$

$f(x_6) = 1.299$

~~Sixth~~

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Sixth app., (n=6)

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$= 1.40$

$f(x_7) = 0.378$

Seventh app., (n=7)

$$x_8 = \frac{x_6 f(x_7) - x_7 f(x_6)}{f(x_7) - f(x_6)} =$$

$$= \frac{(1.458)(0.378) - (1.4)(1.299)}{0.378 - 1.299}$$

$= 1.4$

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$x_3 = 1.875 - \frac{(0.0918)}{\cancel{(6.5469)}}$

$= 1.8609$

0^{th} app., $(n=0)$

$f(x) = x^3 - 4x + 1$
$f'(x) = 3x^2 - 4$
$x_0 = 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1 + \frac{2}{-1}$

$= -1$

first app., $(n=1)$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= -1 - \frac{4}{-1}$

$= 3$

Second app., $(n=2)$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= \cancel{2.5248}$ 2.3043

Third app., $(n=3)$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= \cancel{2.5248 - 2.0609}$

$= 1.9675$

Fourth app., $(n=4)$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$$= 1.8695$$

Fifth app., $(n=5)$

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)}$$

$$= 1.8609$$

Sixth app. $(n=6)$

$$x_7 = x_6 - \frac{f(x_6)}{f'(x_6)}$$

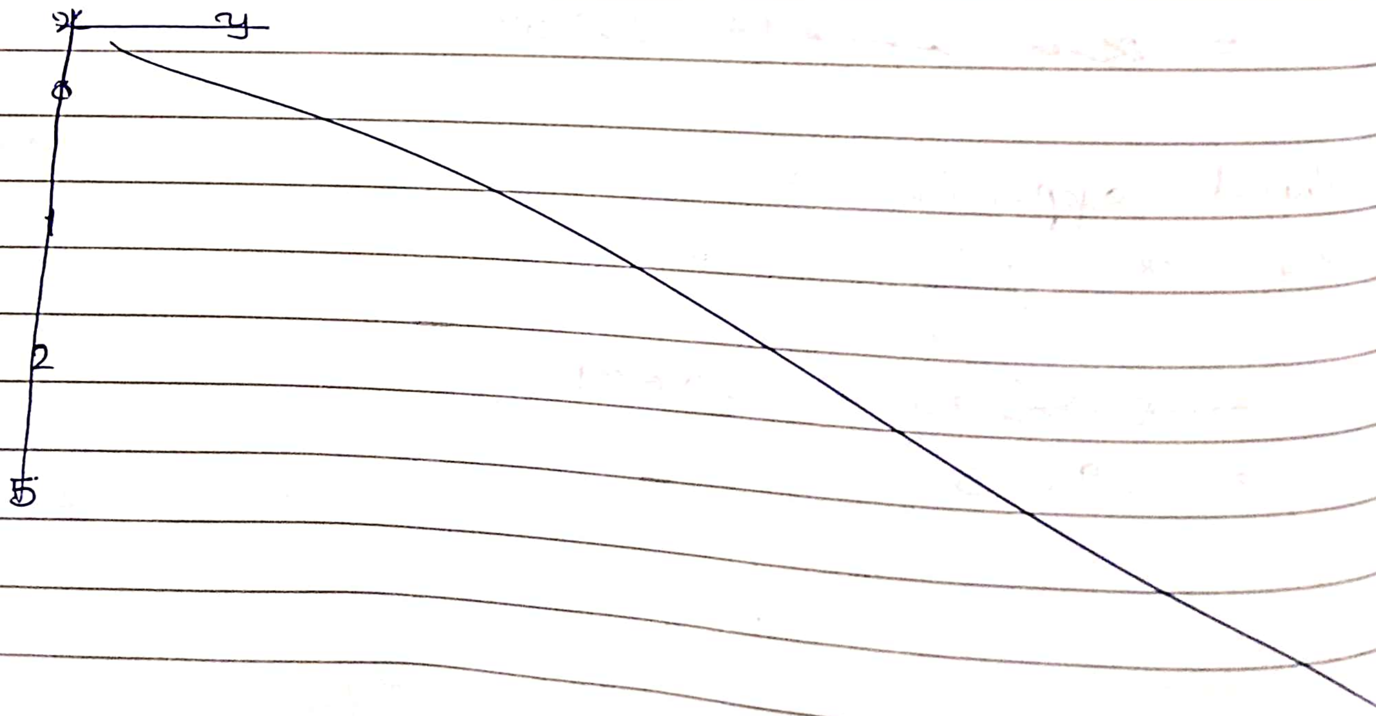
$$= 1.8609.$$

Hence 1.8609 is the real root ~~of~~ correct upto 4 decimal places.

Q3. ~~3~~.

x	0	1	2	5
y	1	3	11	131

Ans.



$x_0 = 0 \qquad y_0 = 1$

$x_1 = 1 \qquad y_1 = 3$

$x_2 = 2 \qquad y_2 = 11$

$x_3 = 5 \qquad y_3 = 131$

$$y = P_n(x) = \sum_{i=0}^{n} L_i(x)\, y_i$$

$$y = L_0(x_0)y_0 + L_1(x_1)y_1 + L_2(x_2)y_2 + L_3(x_3)y_3$$

$$= \cancel{\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_3)}y_0} + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

$$\cancel{=(x-1)(x-2} + (x-$$

$$= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(1) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) +$$

$$\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(11) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(131)$$

$$= \frac{(x-2)(x^2-6x+5)}{-10} + \frac{3x(x^2-7x+10)}{4} +$$

$$\frac{11x(x^2-6x+5)}{6} + \frac{131x(x^2-3x+2)}{60}$$

$$= \frac{x^3-6x^2+5x-2x^2-12x-10}{-10} + \frac{(3x^3-21x^2+30x)}{4} +$$

$$\frac{11x^3-66x^2+55x}{6} + \frac{(131x^3-393x^2+262x)}{60}$$

$$= \frac{(x^3-8x^2-7x-10)}{-10}$$

$$= \frac{(x^3 - 8x^2 + 19x - 12)}{12} + \frac{(3x^3 - 21x^2 + 30x)}{11} +$$

$$\frac{11x^3 - 88x^2 + 155x}{6} + \frac{(131x^3 - 592x^2 + 969x)}{60}$$

$$= \frac{\left(-6x^3 + 48x^2 + 112x + 60 + 45x^3 - 315x^2 + 450x + 110x^3 - 880x^2 + 1550x + 131x^3 - 593x^2 + 969x\right)}{60}$$

$$= \frac{280x^3 - 1330x^2 + 1304x + 60}{60}$$

$$y = \frac{1}{60}(280x^3 - 1330x^2 + 1304x + 60)$$

at $x = 2$, y

$y = -5.8$

Q4.

x	1	2	3	4	5	6
y	6	9	14	21	30	41

find $f(2.5)$

Ans.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	6					
		3				
2	9		2			
		5		0		
3	14		2		0	
		7		0		0
4	21		2		0	
		9		0		
5	30		2			
		11				
6	41					

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$= \cancel{6 +}$$

$$= y_0 + x\Delta y_0 + \frac{x(x-1)}{2!}\Delta^2 y_0 -$$

$$P_n(x) \sum_{i=0}^{n} L_i(x) y_n$$

$$= 6 + 3x + \frac{(x^2 - x)2}{2}$$

$$L_i(x)$$

$$= x^2 - x + 3x + 6$$

$$= x^2 - 2x + 6$$

$$y = (4.5)^2 - 2(4.5) + 6$$

$$= 7.25$$

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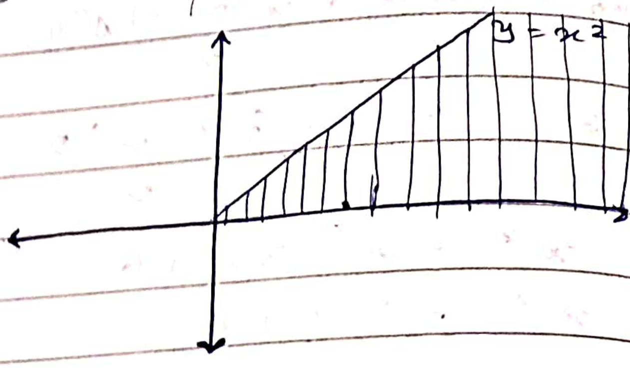
Q. Find the area bounded by the curve $f(x) = x^2$ and the x-axis from $x=0$ to $x=4$ by taking 8 sub-intervals using trapezoidal rule.

Ans. $n = 8$; $a = 0$
$b = 4$



$$h = \frac{b-a}{n}$$

$$h = \frac{4}{8} = 0.5$$

$$I = \int_a^b f(x)\,dx$$

$$= \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})\right]$$

x	0 (x_0)	1/2	1	3/2	2	5/2	3	7/2	4 (x_n)
$y=x^2$	0 (y_0)	1/4 (0.25)	1	9/4 (2.25)	4	25/4 (6.25)	9	12.25	16 (y_n)

$$= \frac{0.5}{2}\left[(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$$

$$= 0.25\left[(0 + 16) + 2(0.25 + 1 + 2.25 + 4 + 6.25 + 9 + 12.25)\right]$$

$$= 21.5$$

To check,

$$I = \int_0^4 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^4 = \frac{64}{3} \approx 21.5$$

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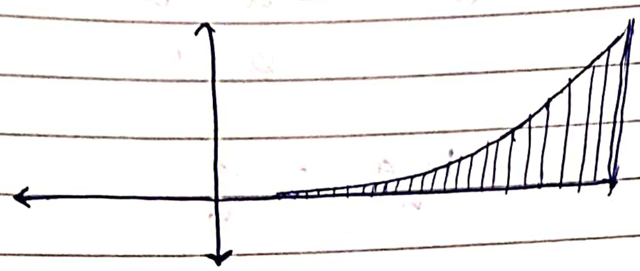
Q. Find the area ~~between~~ bounded by the curve $f(x) = \log_e(x)$ and the x-axis from $x = 1$ to $x = 6$ by taking ~~6~~ ordinates using trapezoidal rule.

Ans. $n = 5$ $\quad h = \frac{b-a}{n} = \frac{6-1}{5} = 1$

$a = 1$

$b = 6$



$$I = \int_a^b f(x)\,dx$$

$\frac{h}{2}[(y_0 +$

x	1	2	3	4	5	6
$\log_e x = y$	0	0.693	1.0986	1.3862	1.6094	1.7917
	y_0	1	2	3	4	y_5

$$I = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4)]$$

$$= 0.5[(0 + 1.7917) + 2(0.693 + 1.0986 + 1.3862 + 1.6094)]$$

$$= 5.6831$$

$$I = \int_a^b f(x)\,dx = \int_1^6 (\log_e x)\,dx$$

$$= [x \ln x - x]_1^6$$

$$= 5.75$$

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Q. Find the area bounded by the curve $f(x) = \frac{1}{1+x}$ and the x-axis from $x = 0$ to 1 taking 7 ordinates, using trap. rule.

Ans. $n = 6$ $\quad a = 0$

$b = 1.$

$$h = \frac{b-a}{n} = \frac{1}{6}$$

x	0	1/6	2/6	1/2	2/3	5/6	1
$y = \frac{1}{1+x}$	1	0.8571	0.75	0.6667	0.6	0.5454	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$I = \int_a^b f(x)\,dx$$

$$= \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{12}\left[(1+0.5) + 2(0.8571+0.75+0.6667+0.6+0.5454)\right]$$

$$= 0.6949$$

Q. Evaluate $\int_0^{0.8}[\log_e(x+1) + \sin(2x)]\,dx$ where x is in radians. Use Simpson's 1/3rd Rule and divide entire interval into 8 equal parts.

Ans. Let $I = \int_0^{0.8}[\log_e(x+1) + \sin(2x)]\,dx$; $n = 8$

$y = \log_e(x+1) + \sin 2x$

$$h = \frac{0.8-}{8} = 0.1$$

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x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
y	0	0.2939	0.5717	0.827	1.0538	1.2469	1.402	1.516	1.5873
	y_0	1	2	3	4	5	6	7	y_8

$$I = \int_a^b f(x)\,dx$$

$$\cancel{\frac{0.1}{2}\left[(y_0+y_8)+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]}$$

$$\cancel{= 0.05\left[(0+1.5873)+2(0.2939+0.5717+0.827+1.0538+1.2469+1.402+1.516)\right]}$$

$$I = \frac{h}{3}\left[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\right]$$

$$= 0.77258$$

Q. Use Simpson's $1/3^{rd}$ rule to obtain $\int_0^{\pi/2} \frac{\sin x}{x}\,dx$ by dividing the interval into 4 parts..

Ans. $n = 4 \qquad a = 0 \;;\; b = \pi/2$

$$h = \frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$$

$$I = \int_a^b f(x)\,dx \qquad y = \frac{\sin x}{x}$$

$\frac{x}{y}$

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
y	1	0.9744	0.9003	0.7842	0.6366
	0	1	2	3	4

$$I = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{\pi}{} \left[(1+0.6366)+4(0.9744+0.7842)+2(0.9003)\right]$$

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$= 1.3707$

Q. Use Simpson's $1/3^{rd}$ rule to evaluate $\int_0^{0.8} e^{-x^2} dx$ taking $h = 0.1$.

Ans

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
y	1	0.99	0.9607	0.9139	08521	0.7788	0.6976	0.6126	0.5272
	0	1	2	3	4	5	6	7	8

$I = \frac{h}{3}[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)]$

$= 0.65764$

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Q. Evaluate $\int_0^{\pi} \frac{\sin^2 x}{5 + 4\cos x} dx$ by Simpson's $3/8$ rule taking $h = \pi/6$.

Ans. $h = \frac{\pi}{6}$ $n = \frac{b-a}{h} = \frac{\pi}{6} \cdot \frac{6}{\pi} = 6$

$a = 0$

$b = \pi$

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
y	0	0.0295	0.1071	0.2	0.25	0.1627	0
	0	1	2	3	4	5	6

$I = \int_0^b f(x)\,dx$

$= \frac{3h}{8}[(y_0 + y_6) + 2(y_3) + 3(y_1 + y_2 + y_4 + y_5)]$

$= \left(\frac{3}{8}\right)\left(\frac{\pi}{6}\right)[(0) + (2)(0.2) + 3(0.0295 + 0.1071 + 0.25 + 0.1627)]$

$= \frac{\pi}{16}[0.4 + 1.6479] = 0.4021$

0.049
0.4021

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Q. Evaluate $\int_{0.2}^{1.4} (e^x + \sin x - \log_e x)\, dx$, by Simpson's 3/8 rule taking $h = 0.1$.

Ans. $a = 0.2$ $h = 0.1$
$b = 1.4$

$n = \frac{1.4 - 0.2}{0.1} = 12$

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
y	3.0295	2.8493	2.7975	2.8212	2.8975	3.0146	3.166	3.3482	3.5597	3.8	4.0698	4.3704	4.7041
	0	1	2	3	4	5	6	7	8	9	10	11	12

$I = \int_a^b f(x)\, dx$

$= \frac{3h}{8} [(y_0 + y_{12}) + 2(y_3 + y_6 + y_9) + 3(y_1 + y_2 + y_4 + y_5 + y_7 + y_8 + y_{10} + y_{11})]$

$= \frac{0.3}{8} [(3.0295 + 4.7041) + 2(2.8212 + 3.166 + 3.8) + 3(2.8493 + 2.7975 + 2.8975 + 3.0146 + 3.3482 + 3.5597 + 4.0698 + 4.3704)]$

$= 4.051$

$$\begin{array}{r} 4.0698 \\ +4.3704 \\ \hline 8.4402 \\ +3.5597 \\ \hline 11.9999 \end{array}$$

Q. Use Simpson's 3/8 rule to evaluate $\int_0^6 x e^x\, dx$ by taking $h = 1.0$.

Ans. $I = \int_0^6 x e^x\, dx = \int_a^b f(x)\, dx$.

$\therefore a = 0$
$b = 6$
$h = 1$
$n = 6$

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x	0	1	2	3	4	5	6
y	0	2.7182	14.778	60.256	218.39	742.06	2420.5

$$I = \int_a^b f(x)\,dx$$

$$= \frac{3h}{8}\left[(y_0+y_6) + 2(y_3) + 3(y_1+y_2+y_4+y_5)\right]$$

$$= \frac{3}{8}\left[(0+2420.5) + 2(60.256) + 3(2.7182 + 14.778 + 218.39 + 742.06)\right]$$

$$= 2053.0689075$$

Q. **Numerical solⁿ of Ordinary Differential Eqⁿs**

1. Modified Euler's Method

Consider the D.E. $\frac{dy}{dx}$

$$\frac{dy}{dx} = f(x,y) \text{ with } y(x_0) = y_0$$

OR

$$x = x_0 ;\ y = y_0$$

- By Euler's Method,

Zeroth approximation (app.),

$$y_1^{(0)} = y_1 = y_0 + hf(x_0, y_0)$$

- By modified Euler's method,

$$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{n})\right] \text{ where } n = 0, 1, 2, \ldots$$

NOTE:- $y_1^{(0)} = y_1$ at 0^{th} app.

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Q. Solve the D.E. $\frac{dy}{dx} = y + xy$; $y(0) = 1$ to find y at $x = 0.2$ using Modified Euler's Method taking $h = 0.2$ correct upto 4 decimal places.

Ans. $\frac{dy}{dx} = y + xy$; $h = 0.2$; $y(0) = 1$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \downarrow \quad \downarrow$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x_0 \quad y_0$

$\Rightarrow f(x, y) = y + xy$; $x_0 = 0$; $y_0 = 1$

$y_1^{(0)} = y_1 = y_0 + h \cdot f(x_0, y_0)$

$= 1 + 0.2 \cdot f(0, 1)$

$= \cancel{2.0} \quad 1 + 0.2 \cdot 1$

$y_1^{(0)} = y_1 = 1.2$

Now $x_1 = x_0 + h$

$= 0 + 0.2 = 0.2$

$\therefore x_1 = 0.2$

By MEM,

$y_1^{(n+1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(n)})]$; $n = 0, 1, 2, 3, \ldots$

$n = 0$

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.2}{2}[f(0, 1) + f(0.2, 1.2)]$

$= 1 + 0.1[(1) + (1.2) + (0.2)(1.2)]$

$= 1.244$

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Now $n=1$,

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(1)})]$

$= 1 + 0.1[(1) + (\cancel{0.0200})(1.244) + (0.2)(1.244)]$

$= 1.2493$

Now $n=2$,

$y_1^{(3)} = 1.2499$

Now $n=3$,

$y_1^{(4)} = 1.2499$

$\therefore y_1 = y(0.2) = 1.2499$

3/5/24 Solve the eq^n $\frac{dy}{dx} = y-x$; $y(0.1) = 2.205$ to find

y at $x=0.2$ using Modified Euler's method taking $h = 0.1$ correct upto 3 decimal places.

Ans $\frac{dy}{dx} = y-x$; $h = 0.1$; $y(0.1) = 2.205$

$x_0 = 0.1$

$y_0 = 2.205$

$f(x,y) = y-x$; $h = 0.\cancel{2}1$; $x_0 = 0.1$; $y_0 = 2.205$

$y_1^0 = y_1 = y_0 + h + f(x_0, y_0)$

$= 2.205 + 0.\cancel{2}1 + (2.205 - 0.1)$

$= 4.41$

$y_1^1 = y_0 + h + f(x_1, y_0)$ $[x_1 = x_0 + h = 0.1 + 0.1$

$= 4.31$

$= 0.2]$

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$\cancel{y_1^2 = y_0 + h \cdot f(x_0, y_0)}$

By MEM,

$$y_1^{(n+1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x, y_1^n)] \quad ; n = 0, 1, 2, \ldots$$

First app., (n=0),

$$y_1^1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^0)]$$

$$= 2.205 + \frac{0.1}{2}[f(0.1, 2.205) + f(0.2, 2.4155)]$$

$$= 2.421025$$

Second app., (n=1)

$$y_1^2 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^1)]$$

$$= 2.42130125$$

Third app., (n=2)

$$y_1^3 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^2)]$$

$$= 2.42130125$$

$\therefore$ The approx. value of y_1 at $x_1 = 0.2$ is 2.42130125.

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Q. Solve the eqⁿ $\frac{dy}{dx} = -xy^2$, $y(0) = 2$, [illegible] y at $x = 0.2$ using MEM taking $h = 0.2$.

Ans $\frac{dy}{dx} = -xy^2$ $x_0 = 0$
$y_0 = 2$
$h = 0.2$.

Euler's method,
$y_1^0 = y_1 = y_0 + h(f(x_0, y_0))$
$= 2 + 0.2(+0)$
$y_1^0 = y_1 = 2$.

$x_1 = x_0 + h$
$= 0 + 0.2$
$= 0.2$.

By MEM,
$$y_1^{(n+1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^n)]$$

First app., $(n=0)$
$$y_1^1 = 2 + \frac{0.2}{2}[(-0) + (0.2)(4)]$$
$= \cancel{0.1 \cdot 0.8 = 0.08}$
$= 1.9200$

Second app., $(n=1)$
$$y_1^2 = 2 + 0.1[(0) + (0.2)(1.92)]$$
$= 1.926272$

Third app., $(n=2)$
$$y_1^3 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^2)]$$

$\therefore y_1^3 = 1.925789.$

Fourth app., $(n=3)$

$y_1^4 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^3)]$

$= 1.925826.$

$\therefore$ The approx. value y_1 at $x_1 = 0.2$ is 1.925.

Q. $\frac{dy}{dx} = \log_e(x+y)$; $y(1) = 2$; $h = 0.2$

Ans. $f(x,y) = \ln(x+y)$.

$x_0 = 1$

$y_0 = 2$

$h = 0.2.$

Euler's method,

$y_1^0 = y_1 = y_0 + h(f(x,y))$

$= 2.21972$

$x_1 = x_0 + h$

$= 1 + 0.2$

$= 1.2$

By MEM, $(n=0)$

$y_1^{n+1} = y_1^1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^n)]$

$= y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^0)]$

$= 2 + \frac{0.2}{2}[(\ln(3)) + (\ln(1.2 + 2.21972))]$

$= 2.232817$

Second app., $(n=1)$

$y_1^2 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^2)]$

$= 2.232817$

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Runge-Kutta 4th Order Method

Consider D.E. $\frac{dy}{dx} = f(x,y)$; $y(x_0) = y_0$

step size $= h$

$k_1 = hf(x_0, y_0)$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$k_1 = hf(x_0, y_0)$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ $\quad k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$

$\therefore$ The value y_1 at x_1 is $y_1 = y_0 + k$

Q. Use 4th order Runge-Kutta method to solve $\frac{dy}{dx} = \log_e(x+y)$; $y(1) = 2$ to calculate y at $x = 1.2$ taking $h = 0.2$.

Ans. $\frac{dy}{dx} = \ln(x+y) = f(x,y)$ $\quad x_0 = 1$

$y_0 = 2$

$h = 0.2$

$x_1 = x_0 + h$

$= 1 + 0.2 = 1.$

$x_1 = 1.2$

$k_1 = 0.2\, f(1,2)$

$= 0.2 \log_e(3)$

$= 0.2197$

$k_1 = hf(x_0, y_0)$

$= 0.2 \cdot (\ln(1+2))$

$= 0.2197224577$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.2 f\left(1.1, 2.1\right)$

$= 0.2 \ln(3.2)$

$= 0.2326$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.2 \ln(1.1 + 2.1)$

$= 0.2 \ln(3.2)$

$= 0.2326$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.2 \ln(1.2, 2.2326)$

$= 0.2 \ln(3.4)$

$= 0.2448$

$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$

$= 0.2325$

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Q1. Find the root of the eq^n $x^2 - 5x + 2 = 0$ between $x = 0$ and $x = 1$ using secant method.

Q2. By Newton-Raphson method. Find the root of the eq^n $e^{-x} - x^2 = 0$ between $x = 0$ and $x = 1$. (4 d.p.)

Q3. Find lagrange's interpolating polynomial passing through a set of points

x	0	1	2
y	1	2	1

Use it to find y at $x = 0.5$ and $\frac{dy}{dx}$ at $x = 1.2$

Q4. Given that:

x	0	1	2	3	4	5
y	1	1	7	25	61	121

Using Newton's interpolating formula, find y at $x = 0.5$.

Q5. Given that:

x	1	2	3	4	5
y	14	30	62	116	198

Using Newton's interpolation formula, find y at $x = 4.5$.

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Q1. $f(x) = x^2 - 5x + 2 = 0$

$x = 0$; $x = 1$

$f(x) = x^2 - 5x + 2$.

$f(0) = 2 > 0$

$f(1) = 1 - 5 + 2 = -2 < 0$. f(

$x_0 = 0$

$x_1 = 1$

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})} \quad ; n = 0, 1, 2, \ldots$$

$n = 1$,

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{0(-2) - 1(2)}{-2 - 2}$$

$$= \frac{-2}{-4} = 0.5.$$

Second app. $(n=2)$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1(-0.25) - (0.5)(-2)}{-0.25 + 2}$$

$= 0.4286$

Third app. $(n=3)$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$= 0.4386$

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Numerical Pr

Q 1 — 10
Q 2 — 10
~~Q 3 — 10~~
Q 4 — 10
Q 5 — 10
Q 6 — 10

Solve any 3

Q
a) 5m
b) 5m
c) 5m
(~~Solve any~~

Q ~~4~~ 5, 6 → Unit V, VI
Q ~~2~~ 3 → Unit IV
Q 1, 2 → Unit I, II, III

Unit I, II, III → 20 marks.
Unit IV → 10 marks
Unit V, VI → 30 marks

classmate
Date
Page

Q. Use secant method to find $f(x) = x - e^{-x}$ correct upto 3 dec. places taking $x_0 = 0$
$x_1 = 1$.

Ans $x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_n)}{f(x_n) - f(x_{n-1})}$

$f(x) = x - e^{-x}$
$f(0) = 1$
$f(1) = 0.36787944$

$x_0 = 0$; $x_1 = 1$
By secant method,

Fourth app. $(n=4)$ $\qquad f(x_4) = -0.0003$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$\cancel{= 0.}$

$= 0.4384$

$\therefore$ Hence the root correct upto 3 decimal places is 0.438.

Q2. $f(x) = e^{-x} - x^2 = 0$ $\qquad x_0 = 0$

$f'(x) = -xe^{-x} - 2x = 0$, $\qquad x_1 = 1$

$f(0) = 1 > 0$ $\qquad f'(0) = -1$

$f(1) = \frac{1}{e} - 1 = -0.632 < 0$ $\qquad f'(1) = -\frac{1}{e} - 2$

By Newton's Raphson Method,

$$x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$$

Zeroth

~~First~~ app., $(n=0)$

$$x_1 = x_0 + \frac{f(x_0)}{f'(x_0)}$$

$$= 0 + \frac{1}{-1}$$

$x_1 = -1$

First app., $(n=1)$

$$x_2 = x_1 + \frac{f(x_1)}{f'(x_1)} = -1 + \frac{(0.632)}{(-2.368)} = -0.7331$$

~~Second app., (n=2)~~

~~$x_3 = x_2 + \frac{f(x_2)}{f'(x_2)}$~~ ~~$f(x_2) = 1.544$~~

~~$f'(x_2) = 0.9858$~~

~~$= -0.7331 + \frac{1.544}{0.9858}$~~

~~$= 0.83314$~~

~~By 1~~

Q2: $f(x) = e^{-x} - x^2 = 0$

$f'(x) = -xe^{-x} - 2x = 0$

$x_0 = 0$

$x_1 = 1$

~~Zeroth app., (n=0)~~ ~~$f(x_0) = +1$~~

~~$f'(x_0) = \cdot 0$~~

~~$x_1 = x_0 + \frac{f(x_0)}{f'(x_0)}$~~

$f(x_1) = \frac{1}{e} - 1 = -0.63212$

$f'(x_1) = -\frac{1}{e} - 2 = -2.367887$

$n = 1,$

$x_2 = x_1 + \frac{f(x_1)}{f'(x_1)} = 1 + \frac{(-0.6321)}{(-2.36787)}$ $f(x_2) = -1.2926$

$f'(x_2) = -2.8907$

$= 1.26695$

$n = 2,$

$x_3 = x_2 + \frac{f(x_2)}{f'(x_2)} = 1.26695 + \frac{1.2926}{2.8907} = 1.7141$

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Q3. $y = P_n(x) = \sum_{i=0}^{n} L_i(x) \cdot y_i$

$x_0 = 0 \qquad y_0 = 1$

$x_1 = 1 \qquad y_1 = 2$

$x_2 = 2 \qquad y_2 = 1$

$$y = L_0(x) y_0 + L_1(x) y_1 + L_2(x) y_2$$

$$= \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(x-1)(x-2)(1)}{(0-1)(0-2)} + \frac{(x-0)(x-2)(2)}{(1-0)(1-2)} + \frac{(x-0)(x-1)(1)}{(2-0)(2-1)}$$

$$= \frac{(x-1)(x-2)}{+2} + \frac{2x(x-2)}{-1} + \frac{x(x-1)}{2}$$

$=$